

$$1) \int_0^1 x 3^x dx$$

$$\begin{aligned} y &= x & dy &= dx \\ du &= 3^x dx & v &= \frac{3^x}{\ln 3} \end{aligned}$$

$$= x \frac{3^x}{\ln 3} \Big|_0^1 - \int_0^1 \frac{3^x}{\ln 3} dx$$

$$\begin{aligned} &= \left(\frac{3}{\ln 3} - 0 \right) - \frac{3^x}{(\ln 3)^2} \Big|_0^1 = \frac{3}{\ln 3} - \left(\frac{3}{(\ln 3)^2} - \frac{1}{(\ln 3)^2} \right) \\ &= \frac{3}{\ln 3} - \frac{2}{(\ln 3)^2} \end{aligned}$$

$$2) \int \frac{x}{6^x} dx = \int x 6^{-x} dx$$

$$\begin{aligned} y &= x & dy &= dx \\ du &= 6^{-x} dx & v &= -\frac{6^{-x}}{\ln 6} \end{aligned}$$

$$= -x \frac{6^{-x}}{\ln 6} + \int \frac{6^{-x}}{\ln 6} dx$$

$$= -\frac{x 6^{-x}}{\ln 6} - \frac{6^{-x}}{(\ln 6)^2} + C = \frac{-x}{6^x \ln 6} - \frac{1}{6^x (\ln 6)^2} + C$$

$$3) \int 2x^2 \sec^2 x \tan x dx$$

$$\begin{aligned} y &= 2x^2 & dy &= 4x dx \\ du &= \sec^2 x \tan x dx & v &= \frac{\tan^2 x}{2} \end{aligned}$$

$$= 2x^2 * \frac{\tan^2 x}{2} - \int \frac{\tan^2 x}{2} (4x) dx$$

$$= x^2 \tan^2 x - \int 2x \tan^2 x dx$$

$$\begin{aligned} y &= 2x & dy &= 2 dx \\ du &= \tan^2 x dx & v &= \tan x - x \end{aligned}$$

$$= x^2 \tan^2 x - (2x(\tan x - x) - \int (\tan x - x) 2 dx)$$

$$= x^2 \tan^2 x - 2x \tan x + 2x^2 - 2 \ln |\cos x| + x^2 + C$$

$$= x^2 \tan^2 x - 2x \tan x - 2 \ln |\cos x| + x^2 + C$$

$$\int 2x^2 \sec^2 x \tan x \, dx$$

خذ السالب

$$y = 2x^2 \quad dy = 4x \, dx$$

$$du = \sec^2 x \tan x \, dx \quad u = \frac{\tan^2 x}{2}$$

$$= x^2 \tan^2 x - \int 4x \frac{\tan^2 x}{2} \, dx$$

$$= x^2 \tan^2 x - \left(2x(\tan x - x) - \int (2 \tan x - 2x) \, dx \right)$$

$$y = 2x \quad dy = 2 \, dx$$

$$du = \tan^2 x \, dx \quad u = \tan x - x$$

$$= x^2 \tan^2 x - 2x \tan x + x^2 - 2 \ln |\cos x| - x^2 + C$$

$$= x^2 \tan^2 x - 2x \tan x - 2 \ln |\cos x| + x^2 + C$$

$$= x^2 \tan^2 x + x^2 - 2x \tan x - 2 \ln |\cos x| + C$$

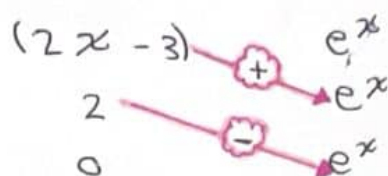
$$= x^2 (\tan^2 x + 1) - 2x \tan x - 2 \ln |\cos x| + C$$

$$= x^2 \sec^2 x - 2x \tan x - 2 \ln |\cos x| + C$$

حجوة أخرى للإجابة

$$1) \int e^x(2x-3) dx$$

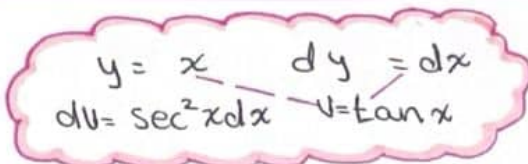
$$= (2x-3)e^x - 2e^x + c$$



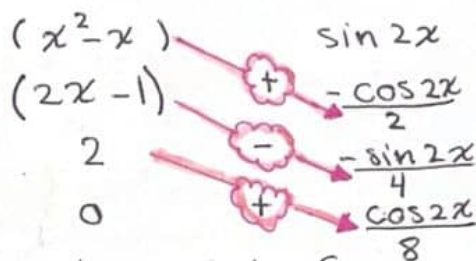
$$2) \int x \sec^2 x dx$$

$$= x \tan x - \int \tan x dx$$

$$= x \tan x + \ln |\cos x| + c$$



$$3) \int (x^2 - x) \sin 2x dx$$



$$-\frac{(x^2-x)}{2} \cos 2x + \frac{(2x-1)}{4} \sin 2x + \frac{1}{4} \cos 2x + c$$

$$4) \int_0^{\pi/2} e^x \cos x dx$$

$$y = \cos x \quad dy = -\sin x dx$$

$$dv = e^x dx \quad v = e^x$$

$$= \cos x (e^x) \Big|_0^{\pi/2} + \int_0^{\pi/2} \sin x (e^x) dx$$

$$y = \sin x \quad dy = \cos x dx$$

$$= (0-1) + (e^x \sin x \Big|_0^{\pi/2} - \int_0^{\pi/2} e^x \cos x dx) \quad dv = e^x dx \quad v = e^x$$

$$2 \int_0^{\pi/2} e^x \cos x dx = -1 + (e^{\pi/2} - 0) = e^{\pi/2} - 1$$

$$\int_0^{\pi/2} e^x \cos x dx = \frac{e^{\pi/2} - 1}{2}$$

$$5) \int \frac{e^{-x}}{\csc 2x} dx = \int e^{-x} \sin 2x dx$$

$$= -e^{-x} \sin 2x + \int 2e^{-x} \cos 2x dx$$

$$= -e^{-x} \sin 2x + 2e^{-x} \cos 2x - \int 4e^{-x} \sin 2x dx$$

$$5 \int e^{-x} \sin 2x dx = -e^{-x} \sin 2x - 2e^{-x} \cos 2x$$

$$\int e^{-x} \sin 2x dx = -\frac{e^{-x} \sin 2x}{5} - \frac{2}{5} e^{-x} \cos 2x + C$$

$$y = \sin 2x \quad dy = 2 \cos 2x dx$$

$$dv = e^{-x} dx \quad v = -e^{-x}$$

$$y = 2 \cos 2x \quad dy = -4 \sin 2x dx$$

$$dv = e^{-x} dx \quad v = -e^{-x}$$

$$6) \int \csc^3 x dx = \int \csc^2 x \csc x dx$$

$$= -\csc x \cot x - \int \csc x \cot^2 x dx$$

$$= -\csc x \cot x - \int \csc x (\csc^2 x - 1) dx$$

$$\int \csc^3 x dx = -\csc x \cot x - \int \csc^3 x dx + \int \csc x dx$$

$$2 \int \csc^3 x dx = -\csc x \cot x + -\ln |\csc x + \cot x|$$

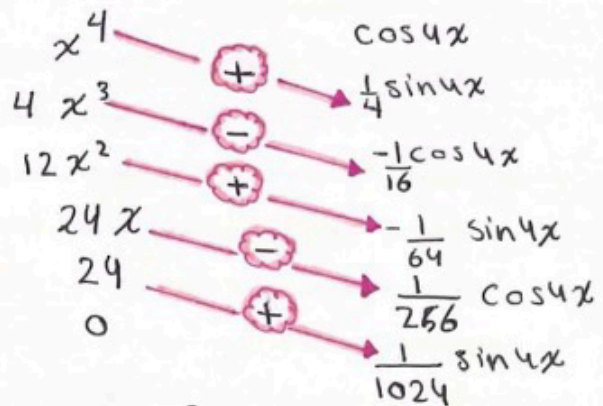
$$\int \csc^3 x dx = \frac{-1}{2} (\csc x \cot x + \ln |\csc x + \cot x|) + C$$

$$y = \csc x \quad dy = -\csc x \cot x dx$$

$$dv = \csc^2 x dx \quad v = -\cot x$$

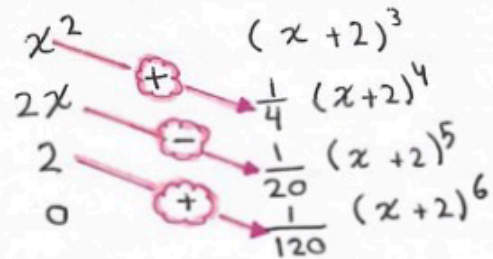
$$\frac{\csc x + \cot x}{\csc x + \cot x}$$

$$1) \int x^4 \cos 4x \, dx$$



$$= \frac{x^4}{4} \sin 4x + \frac{x^3}{4} \cos 4x - \frac{3}{16} x^2 \sin 4x - \frac{3}{32} x \cos 4x + \frac{3}{128} \sin 4x + C$$

$$2) \int x^2 (x+2)^3 \, dx$$



$$= \frac{x^2}{4} (x+2)^4 - \frac{x}{10} (x+2)^5 + \frac{1}{60} (x+2)^6 + C$$

$$1) \int \sin \sqrt{x} \, dx$$

$$= \int \sin u \, (2u \, du)$$

$$= \int 2u \sin u \, du$$

$$= -2u \cos u + 2 \sin u + C$$

$$= -2\sqrt{x} \cos \sqrt{x} + 2 \sin \sqrt{x} + C$$

$$u = \sqrt{x} \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}} \rightarrow dx = 2\sqrt{x} \, du$$



$$2) \int x^3 \cos^2 x^2 \, dx$$

$$= \int \frac{x^2}{2} \cos^2 u \, du$$

$$= \int \frac{u}{4} (1 + \cos 2u) \, du$$

$$= \int \left(\frac{u}{4} + \frac{u \cos 2u}{4} \right) \, du$$

$$= \frac{u^2}{8} + \frac{u \sin 2u}{8} - \int \frac{\sin 2u}{8} \, du$$

$$= \frac{u^2}{8} + \frac{u \sin 2u}{8} + \frac{\cos 2u}{16} + C$$

$$= \frac{x^4}{8} + \frac{x^2 \sin 2x^2}{8} + \frac{\cos 2x^2}{16} + C$$

$$u = x^2 \quad \frac{du}{2x} = dx$$

$$y = \frac{u}{4} \quad dy = \frac{du}{4}$$

$$dv = \cos 2u \, du \quad v = \frac{\sin 2u}{2}$$

في الوقت متسع وفيك عزيمة
والدرب ممتد... وانت شباب



$$3) \int \frac{5^{\frac{1}{x^2}}}{x^5} dx$$

* الحل : تعويض ثم أجزاء

$$\int \frac{5^y}{x^5} \cdot \frac{-x^3}{2} dy$$

$$= \int -\frac{1}{2} 5^y \cdot \frac{1}{x^2} dy$$

نعود للخط

$$= \int -\frac{1}{2} 5^y \cdot y dy$$

$$\int \underbrace{-\frac{1}{2} y}_v \cdot \underbrace{5^y}_{du} dy$$

$$v = -\frac{1}{2} y \longrightarrow du = -\frac{1}{2}$$

$$\int du = \int 5^y \longrightarrow u = \frac{5^y}{\ln 5}$$

$$= \frac{-y}{2 \ln 5} 5^y - \int -\frac{1}{2 \ln 5} 5^y dy$$

$$= \frac{-y}{2 \ln 5} 5^y + \frac{1}{2 \ln 5} \cdot \frac{5^y}{\ln 5} + C$$

و نسي جزء y و نسي C

$$y = \frac{1}{x^2}$$

$$\frac{dy}{dx} = \frac{-1(2x)}{(x^2)^2}$$

$$\frac{dy}{dx} = \frac{-2x}{x^4}$$

$$\frac{-x^3}{2} dy = dx$$

$$= \frac{-\frac{1}{x^2}}{2 \ln 5} 5^{\frac{1}{x^2}} + \frac{5^{\frac{1}{x^2}}}{2(\ln 5)^2} + C$$

$$= \frac{-5^{\frac{1}{x^2}}}{2 \ln 5 x^2} + \frac{5^{\frac{1}{x^2}}}{2(\ln 5)^2} + C$$

$$1) \int_0^{1/2} \frac{dx}{1-x^2}$$

$$= \int_0^{1/2} \left(\frac{1}{2} \cdot \frac{1}{1-x} dx \right) + \int_0^{1/2} \left(\frac{1}{2} \cdot \frac{1}{1+x} dx \right)$$

$$= \frac{-1}{2} \ln |1-x| \Big|_0^{1/2} + \frac{1}{2} \ln |1+x| \Big|_0^{1/2}$$

$$= \frac{-1}{2} \left(\ln \frac{1}{2} - \ln 1 \right) + \frac{1}{2} \left(\ln \frac{3}{2} - \ln 1 \right)$$

$$= \frac{1}{2} \ln 2 + \frac{1}{2} \ln \frac{3}{2} = \frac{1}{2} \ln \left(2 \cdot \frac{3}{2} \right) = \frac{1}{2} \ln 3 = \ln \sqrt{3}$$

$$\frac{1}{1-x^2} = \frac{A}{1-x} + \frac{B}{1+x}$$

$$1 = A(1+x) + B(1-x)$$

$x=1 \rightarrow 1 = 2A \rightarrow A = \frac{1}{2}$

$x=-1 \rightarrow 1 = 2B \rightarrow B = \frac{1}{2}$

$$2) \int_0^4 \frac{5x+5}{x^2+x-6} dx$$

$$= \int_0^4 \frac{2}{x+3} dx + \int_0^4 \frac{3}{x-2} dx$$

$$= 2 \ln |x+3| \Big|_0^4 + 3 \ln |x-2| \Big|_0^4$$

$$= 2(\ln 7 - \ln 3) + 3(\ln 2 - \ln 2)$$

$$= 2 \ln \frac{7}{3} = \ln \frac{49}{9}$$

$$\frac{5x+5}{(x+3)(x-2)} = \frac{B}{x+3} + \frac{A}{x-2}$$

$$5x+5 = B(x-2) + A(x+3)$$

$x=2 \rightarrow 15 = 5A \rightarrow A = 3$

$x=-3 \rightarrow -10 = -5B \rightarrow B = 2$