واجب 1
ص 32

1) $\frac{1}{1-\cos x} * \frac{1+\cos x}{1+\cos x}$

$$\frac{1+\cos x}{(1-\cos x)(1+\cos x)} = \frac{1+\cos x}{1-\cos^2 x}$$

$$= \frac{1+\cos x}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x} + \frac{\cos x}{\sin^2 x}$$

$$= \csc^2 x + \cot x \csc x$$

حل اخر غريب

$$\frac{1}{1-\cos x} = \frac{1}{2\sin^2 \frac{x}{2}}$$

$$= \frac{1}{2}\csc^2 \frac{x}{2}$$

$\sin^2 \frac{x}{2} = \frac{1}{2}(1-\cos x)$

$2\sin^2 \frac{x}{2} = 1-\cos x$

2) $\frac{1}{1-\sec x} * \frac{1+\sec x}{1+\sec x}$

$$= \frac{1+\sec x}{(1-\sec x)(1+\sec x)} = \frac{1+\sec x}{1-\sec^2 x} = \frac{1+\sec x}{-\tan^2 x}$$

$$= \frac{-1}{\tan^2 x} + \frac{\sec x}{\tan^2 x} = -\cot^2 x + \sec x \cot^2 x$$

واجب
ص 33

اثبت $\frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$

حل (1) الطرف اليمين

$$= \frac{\sin x}{1+\cos x} * \frac{1-\cos x}{1-\cos x}$$

$$= \frac{\sin x(1-\cos x)}{1-\cos^2 x}$$

$$= \frac{\sin x(1-\cos x)}{\sin^2 x}$$

$$= \frac{1-\cos x}{\sin x} \quad \#\#$$

حل (2) الطرف اليسار

$$\frac{1-\cos x}{\sin x} * \frac{1+\cos x}{1+\cos x}$$

$$= \frac{1-\cos^2 x}{\sin x(1+\cos x)} = \frac{\sin^2 x}{\sin x(1+\cos x)}$$

$$= \frac{\sin x}{1+\cos x} \quad \#\#$$

واجب
ص 36

1) $\dfrac{(\sin x+\cos x)^2}{\sin^2 x-\cos^2 x} = \dfrac{\sin^2 x-\cos^2 x}{(\sin x-\cos x)^2}$

الطرف اليسار

$$\frac{(\sin x+\cos x)^2}{(\sin x-\cos x)(\sin x+\cos x)} = \frac{\sin x+\cos x}{\sin x-\cos x} * \frac{\sin x-\cos x}{\sin x-\cos x}$$

$$= \frac{\sin^2 x-\cos^2 x}{(\sin x-\cos x)^2}$$

2)
$$\frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}=0$$

$$\frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}+\frac{\sin B\cos C-\cos B\sin C}{\cos B\cos C}+\frac{\sin C\cos A-\cos C\sin A}{\cos C\cos A}$$

$$=\frac{\sin A\cos B}{\cos A\cos B}-\frac{\cos A\sin B}{\cos A\cos B}+\frac{\sin B\cos C}{\cos B\cos C}-\frac{\cos B\sin C}{\cos B\cos C}+\frac{\sin C\cos A}{\cos C\cos A}-\frac{\cos C\sin A}{\cos C\cos A}$$

$$=\tan A-\tan B+\tan B-\tan C-\tan C-\tan A$$

$$=0$$

3) $\cos 2x+2\cos x+1 = 2\cos x(\cos x+1)$

حل الطرف اليمين

$$=2\cos^2 x+2\cos x$$

$(1+\cos 2x)$

$$=1+\cos 2x+2\cos x$$

$$=\cos 2x+2\cos x+1$$

حل الطرف اليسار

$$=(2\cos^2 x-1)+2\cos x+1$$

$$=2\cos^2 x+2\cos x$$

$$=2\cos x(\cos x+1)$$

واجب
ص 34

4) $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$

الطرف اليسار

$$\frac{2\tan x}{\sec^2 x} = 2\tan x \cos^2 x$$

$$= 2\frac{\sin x}{\cos x}\cos^2 x$$

$$= 2\sin x \cos x$$

$$= \sin 2x$$

5) $\tan y = \frac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)}$

الطرف اليمين

$$= \frac{2\cos\left(\frac{x+y+(x-y)}{2}\right)\sin\left(\frac{x+y-(x-y)}{2}\right)}{2\cos\left(\frac{x+y+(x-y)}{2}\right)\cos\left(\frac{x+y-(x-y)}{2}\right)}$$

$$= \frac{\cos\frac{2x}{2}\sin\frac{2y}{2}}{\cos\frac{2x}{2}\cos\frac{2y}{2}} = \frac{\sin y}{\cos y} = \tan y$$

6) $\ln|\sec x| = -\ln|\cos x|$

حل ② الطرف اليمين

$$\ln|\sec x| = \ln\left|\frac{1}{\cos x}\right|$$

$$= \ln|\cos^{-1} x|$$

$$= -\ln|\cos x|$$

حل ① الطرف اليسار

$$-\ln|\cos x| = \ln|\cos^{-1} x|$$

$$= \ln\left|\frac{1}{\cos x}\right|$$

$$= \ln|\sec x|$$

$f(x) = \sin x$

$g(x) = \cos x$

$\sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = \frac{-1}{\sqrt{5}}$

$\sin\alpha = -\frac{\sqrt{15}}{4}$, $\cos\alpha = -\frac{1}{4}$

1) $g(\alpha+\theta) = \cos(\alpha+\theta) = \cos\alpha\cos\theta - \sin\alpha\sin\theta$

$= \left(\frac{-1}{4}\right)\left(\frac{-1}{\sqrt{5}}\right) - \left(\frac{-\sqrt{15}}{4}\right)\left(\frac{2}{\sqrt{5}}\right)$

$= \frac{1}{4\sqrt{5}} + \frac{2\sqrt{15}}{4\sqrt{5}} = \frac{1+2\sqrt{15}}{4\sqrt{5}}$

2) $f\left(\frac{\theta}{2}\right) = \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1+\frac{1}{\sqrt{5}}}{2}}$

$= \sqrt{\frac{\sqrt{5}+1}{2\sqrt{5}}}$