

$$\int 4 \cdot \frac{\cos 2x}{\sin 2x} \cdot \frac{1}{\sin 2x} \cdot dx$$

$$4 \int \cot 2x \cdot \csc 2x \cdot dx$$

$$= 4 \left(-\frac{\csc 2x}{2} \right) + C$$

$$= -2 \csc 2x + C$$

$$\textcircled{4} \int e^{2 \ln(1 + \sin x)} \cdot dx$$

$$\int e^{\ln(1 + \sin x)^2} \cdot dx$$

$$\int (1 + \sin x)^2 \cdot dx$$

$$\int (1 + 2 \sin x + \sin^2 x) \cdot dx$$

$$\int \left(1 + 2 \sin x + \frac{1}{2} (1 - \cos 2x) \right) \cdot dx$$

$$= x - 2 \cos x + \frac{1}{2} \left(x - \frac{\sin 2x}{2} \right) + C^2$$

$$\textcircled{5} \int \frac{\cos^2 x}{1 - \sin x} \cdot dx$$

تحويل : جد *

$$\cos^2 x = 1 - \sin^2 x$$

$$\int \frac{1 - \sin^2 x}{1 - \sin x} \cdot dx$$

$$\int \frac{(1 - \sin x)(1 + \sin x)}{1 - \sin x} \cdot dx$$

$$\int (1 + \sin x) \cdot dx = x - \cos x + C$$

* تفصيل :-

$$\textcircled{1} \int \frac{\cos x^3 - 5}{1 - \sin^2 x} \cdot dx$$

$$1 - \sin^2 x = \cos^2 x \quad \therefore \text{جد} *$$

$$\int \frac{\cos x^3 - 5}{\cos^2 x} \cdot dx$$

$$\int \left(\frac{\cos^3 x}{\cos^2 x} - \frac{5}{\cos^2 x} \right) \cdot dx$$

$$\int (\cos x - 5 \sec^2 x) \cdot dx$$

$$= \sin x - 5 \tan x + C$$

$$\textcircled{2} \int \sec x (\tan x + \cos x) \cdot dx$$

$$\int (\sec x \tan x + \sec x \cdot \cos x) \cdot dx$$

$$\int \left(\sec x \tan x + \frac{1}{\cancel{\cos x}} \cdot \cancel{\cos x} \right) \cdot dx$$

$$\int (\sec x \tan x + 1) \cdot dx$$

$$= \sec x + x + C$$

$$\textcircled{3} \int \frac{\cos 2x}{\sin^2 x \cos^2 x} \cdot dx$$

$$\int \frac{\cos 2x}{(\sin x \cos x)^2} \cdot dx \quad \therefore \text{جد} *$$

$$\int \frac{\cos 2x}{\left(\frac{1}{2} \sin 2x\right)^2} \cdot dx$$

$$\int \frac{\cos 2x}{\frac{1}{4} \sin^2 2x} \cdot dx$$

* واجب مفصلة (12)

$$\int \sin^4 x \, dx$$

* الحل :- ان الراس في هذه متباينات

$$\int (\sin^2 x)^2 \, dx$$

$\hookrightarrow \left(\frac{1 - \cos 2x}{2}\right)$ تباينات
من

$$\int \left(\frac{1}{2}(1 - \cos 2x)\right)^2 \, dx$$

فك الشرح

$$\int \frac{1}{4}(1 - 2\cos 2x + \cos^2 x) \, dx$$

$\left(\frac{1 + \cos 4x}{2}\right)$ تباينات
من

$$\frac{1}{4} \int 1 - 2\cos 2x + \frac{1}{2}(1 + \cos 4x) \, dx$$

$$= \frac{1}{4} \left[x - \frac{2\sin 2x}{2} + \frac{1}{2} \left(x + \frac{\sin 4x}{4} \right) \right] + C$$

$$= \frac{1}{4} \left[x - \sin 2x + \frac{1}{2}x + \frac{\sin 4x}{8} \right] + C$$

$$= \frac{1}{4} \left[\frac{3}{2}x - \sin 2x + \frac{\sin 4x}{8} \right] + C$$

$$= \frac{3}{8}x - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C_1$$

* حل آخر لسؤال (3) :-

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cos^2 x} \cdot dx$$

$$\int \frac{\cos^2 x}{\sin^2 x \cos^2 x} - \frac{\sin^2 x}{\sin^2 x \cos^2 x} \cdot dx$$

$$\int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} \cdot dx$$

$$\int (\csc^2 x - \sec^2 x) \cdot dx$$

$$= -\cot x - \tan x + C$$

* حل آخر لسؤال (5) :

اخرى بالجراف

$$\int \frac{\cos^2 x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} \cdot dx$$

$$\int \frac{\cos^2 x (1 + \sin x)}{1 - \sin^2 x} \cdot dx$$

$$\int \frac{\cos^2 x (1 + \sin x)}{\cos^2 x} \cdot dx$$

$$\int (1 + \sin x) \cdot dx$$

$$= x - \cos x + C$$

$$\int \sin^4 x \cos^4 x \, dx$$

- : حل

$$= \int (\sin x \cos x)^4 \, dx$$

↳ ابدأ

$$= \int \left(\frac{1}{2} \sin 2x\right)^4 \, dx$$

$$= \int \frac{1}{16} \sin^4 2x \, dx$$

$$= \frac{1}{16} \int (\sin^2 2x)^2 \, dx$$

↳ ابدأ

$$= \frac{1}{16} \int \left(\frac{1 - \cos 4x}{2}\right)^2 \, dx$$

$$= \frac{1}{16} \int \frac{1}{4} (1 - 2 \cos 4x + \cos^2 4x) \, dx$$

↳ ابدأ

$$= \frac{1}{64} \int 1 - 2 \cos 4x + \frac{1}{2} (1 + \cos 8x) \, dx$$

$$= \frac{1}{64} \left[x - \frac{2 \sin 4x}{4} + \frac{1}{2} \left(x + \frac{\sin 8x}{8} \right) \right] + C$$

$$= \frac{1}{64} \left[x - \frac{\sin 4x}{2} + \frac{1}{2} x + \frac{\sin 8x}{16} \right] + C$$

$$= \frac{1}{64} \left[\frac{3}{2} x - \frac{\sin 4x}{2} + \frac{\sin 8x}{16} \right] + C$$

$$\frac{3}{128} x - \frac{\sin 4x}{128} + \frac{\sin 8x}{1024} + C$$

* تمرين - صيغة (13) :-

$$\int \frac{1 - \sin x}{\sin \frac{x}{2} - \cos \frac{x}{2}} dx$$

* الحل : فك البسط
التوضيح

مباشرة
[العنايه]

$$\begin{array}{ccc} 1 & - & \sin x \\ \downarrow & & \downarrow \text{(الاجب)} \\ \sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} & & 2 \sin \frac{x}{2} \cos \frac{x}{2} \end{array}$$

$$\begin{aligned} 1 - \sin x &= \sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2} \\ &= \sin^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2} + \cos^2 \frac{x}{2} \\ &= (\sin \frac{x}{2} - \cos \frac{x}{2}) (\sin \frac{x}{2} + \cos \frac{x}{2}) \\ &= (\sin \frac{x}{2} - \cos \frac{x}{2})^2 \end{aligned}$$

$$\begin{aligned} &= \int \frac{(\sin \frac{x}{2} - \cos \frac{x}{2})^2}{\cancel{\sin \frac{x}{2} - \cos \frac{x}{2}}} dx \\ &= \int (\sin \frac{x}{2} - \cos \frac{x}{2}) dx \\ &= \frac{-\cos \frac{x}{2}}{\frac{1}{2}} - \frac{\sin \frac{x}{2}}{\frac{1}{2}} + C \\ &= -2 \cos \frac{x}{2} - 2 \sin \frac{x}{2} + C \end{aligned}$$

$$\int \frac{1}{\sec x - 1} dx \times \frac{\sec x + 1}{\sec x + 1}$$

* الحل : ضرب بالمرافق

$$= \int \frac{\sec x + 1}{\sec^2 x - 1} dx$$

↳ tan²x

$$= \int \frac{\sec x + 1}{\tan^2 x} dx = \int \frac{\sec x}{\tan^2 x} dx + \int \frac{1}{\tan^2 x} dx$$

$$= \int \frac{\frac{1}{\cos x}}{\frac{\sin^2 x}{\cos^2 x}} dx + \int \cot^2 x dx = \int \frac{\cos x}{\sin^2 x} dx + \int (\csc^2 x - 1) dx$$

[الغني ولفقر]

$$= \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} dx + (-\cot x - x) + C$$

$$= \int \cot x \csc x dx - \cot x - x + C$$

$$= -\csc x - \cot x - x + C$$

$$\int \frac{1}{1 - \csc x} dx = \frac{1 + \csc x}{1 + \csc x}$$

$$= \int \frac{1 + \csc x}{1 - \csc^2 x} dx$$

$$= \int \frac{1 + \csc x}{-\cot^2 x} dx$$

$$= \int \frac{-1}{\cot^2 x} dx - \int \frac{\csc x}{\cot^2 x} dx$$

$$= \int -\tan^2 x dx - \int \frac{\frac{1}{\sin x}}{\frac{\cos^2 x}{\sin x}} dx$$

$$= -\int (\sec^2 x - 1) dx - \int \frac{\sin x}{\cos^2 x} dx$$

[الغني والفقيه]

$$= -(\tan x - x) - \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$$

$$= -\tan x + x - \int \tan x \sec x dx$$

$$= -\tan x + x - \sec x + C$$

* ترمين ————— فترات (1u) :-

$$\int \frac{\cos 3x}{\cos x} = \int \frac{\overset{\text{سین کون}}{\cos(2x+x)}}{\cos x} dx$$

$$= \int \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x} dx$$

$$= \int \frac{\cos 2x \cancel{\cos x} - \sin 2x \cancel{\sin x}}{\cancel{\cos x}} dx = \int \frac{2 \sin x \cancel{\cos x} \sin x}{\cancel{\cos x}} dx$$

$$= \int \cos 2x dx - \int 2 \sin^2 x dx$$

$$= \frac{\sin 2x}{2} - 2 \int \frac{1}{2} (1 - \cos 2x) dx$$

$$= \frac{\sin 2x}{2} - 2 \cdot \frac{1}{2} \left(x - \frac{\sin 2x}{2} \right) + C$$

$$= \frac{\sin 2x}{2} - x + \frac{\sin 2x}{2} = \sin 2x - x + C$$

حل المسائل

3 $\int \sqrt{\frac{2x+1}{x}} \cdot \frac{1}{x^2} \cdot dx$

* الحل: نجعل ثم تعويض

$$\int \sqrt{\frac{2x+1}{x}} \cdot \frac{1}{x^2} \cdot dx$$

$$\int \sqrt{2 + \frac{1}{x}} \cdot \frac{1}{x^2} \cdot dx$$

$$\int u \cdot \frac{1}{x^2} \cdot -2u x \cdot du$$

$$\int -2u^2 \cdot du$$

$$= -2 \frac{u^3}{3} + C$$

$$= -\frac{2}{3} \left(\sqrt{2 + \frac{1}{x}} \right)^3 + C$$

$$\begin{aligned} u &= \sqrt{2 + \frac{1}{x}} \\ u^2 &= 2 + \frac{1}{x} \\ 2u \cdot \frac{du}{dx} &= \frac{-1}{x^2} \\ -2u x^2 \cdot du &= dx \end{aligned}$$

4 $\int_0^2 \frac{x^3}{\sqrt{(x^2+9)^3}} \cdot dx$

* الحل: نجعل ثم تعويض

$$\int_0^2 \frac{x^3}{(x^2+9)^{\frac{3}{2}}} \cdot dx =$$

$$= \int_0^2 x^3 (x^2+9)^{-\frac{3}{2}} \cdot dx$$

$$\int_9^{13} x^{\frac{3}{2}} (u)^{-\frac{3}{2}} \cdot \frac{du}{2x}$$

$$\int_9^{13} \frac{1}{2} (u-9) \cdot u^{-\frac{3}{2}} \cdot du$$

$$\begin{aligned} u &= x^2 + 9 \\ \frac{du}{dx} &= 2x \\ \frac{du}{2x} &= dx \end{aligned}$$

* تم الحل

1 $\int \frac{\cot^2 \frac{1}{x}}{x^2} \cdot dx$

* الحل:

$$\int \frac{\cot^2 u}{x^2} \cdot -x^2 \cdot du$$

$$\int -\cot^2 u \cdot du$$

$$\int -(csc^2 u - 1) \cdot du$$

$$\int (1 - csc^2 u) \cdot du$$

$$= u + \cot u + C$$

$$= \frac{1}{x} + \cot \frac{1}{x} + C$$

$$\begin{aligned} u &= \frac{1}{x} \\ \frac{du}{dx} &= \frac{-1}{x^2} \\ -x^2 \cdot du &= dx \end{aligned}$$

2 $\int \frac{(5 + \sqrt[3]{x})^7}{\sqrt[3]{x^2}} \cdot dx$

* الحل:

$$\int \frac{(5+u)^7}{u^2} \cdot 3u^2 \cdot du$$

$$\int 3(5+u)^7 \cdot du$$

$$3 \frac{(5+u)^8}{8} + C$$

$$\frac{3}{8} (5 + \sqrt[3]{x})^8 + C$$

$$\begin{aligned} u &= \sqrt[3]{x} \\ u^3 &= x \\ 3u^2 \cdot \frac{du}{dx} &= 1 \\ 3u^2 \cdot du &= dx \end{aligned}$$

$$\textcircled{6} \int (x-1) e^{(x-1)^2} dx$$

* الحل : تعويض (قوة الأسّي)

$$\int_1^1 \cancel{(x-1)} e^u \cdot \frac{du}{2 \cancel{(x-1)}}$$

$$\int_1^1 \frac{1}{2} e^u \cdot du$$

$$u = (x-1)^2$$

$$\frac{du}{dx} = 2(x-1)$$

$$\frac{du}{2(x-1)} = dx$$

الاجواب

Zero

تبدل الحدود

$$x=2 \rightarrow u=1$$

$$x=0 \rightarrow u=1$$

لقد تجاوزتم الكثير

لن تهزمكم الخطوة المتبقية

انتوا_قدها

* تكملة حل سؤال (4) :-

$$\int_9^{13} \frac{1}{2} (u^{-\frac{1}{2}} - 9u^{-\frac{3}{2}}) \cdot dx$$

$$\frac{1}{2} \left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \Big|_9^{13} - 9 \frac{u^{-\frac{1}{2}}}{-\frac{1}{2}} \Big|_9^{13} \right)$$

$$\frac{1}{2} \left(2\sqrt{u} \Big|_9^{13} + \frac{18}{\sqrt{u}} \Big|_9^{13} \right)$$

$$\sqrt{u} \Big|_9^{13} + \frac{9}{\sqrt{u}} \Big|_9^{13}$$

$$= \sqrt{13} - \sqrt{9} + 9 \left(\frac{1}{\sqrt{13}} - \frac{1}{\sqrt{9}} \right)$$

$$= \sqrt{13} - 3 + \frac{9}{\sqrt{13}} - 3$$

$$= \sqrt{13} + \frac{9}{\sqrt{13}} - 6$$

$$\textcircled{5} \int \frac{\sin x \sqrt{\sin^2 x + 4}}{\sec x} \cdot dx$$

* الحل :- نجعل ثم تعويض

$$\int \sin x \cos x \sqrt{(\sin^2 x + 4)} \cdot dx$$

$$\int \cancel{\sin x} \cos x u \cdot \frac{u \cdot du}{\cancel{\sin x} \cos x}$$

$$\int u^2 \cdot du$$

$$\frac{u^3}{3} + C$$

$$\frac{(\sqrt{\sin^2 x + 4})^3}{3} + C$$

$$u = \sqrt{\sin^2 x + 4}$$

$$u^2 = \sin^2 x + 4$$

$$2u \cdot \frac{du}{dx} = 2 \sin x \cos x$$

$$\frac{u \cdot du}{\sin x \cos x} = dx$$

$$\frac{1}{5} \cdot \frac{u^{\frac{4}{3}}}{\frac{4}{3}} + C$$

$$\frac{1}{5} \cdot \frac{3}{4} (\sqrt[3]{u})^4 + C$$

$$\frac{3}{20} (\sqrt[3]{7 + \tan x})^4 + C$$

2

$$\int \frac{\sqrt[3]{x}}{5 - x\sqrt[3]{x}} \cdot dx$$

* الحل : تعويض

$$\int \frac{u}{5 - x\sqrt[3]{x}} \cdot 3u^2 \cdot du$$

$$\int \frac{3u^3}{5 - u^3 \cdot u} \cdot du$$

$$\int \frac{3u^3}{5 - u^4} \cdot du$$

$$\frac{3}{-4} \int \frac{-4 \cdot u^3}{5 - u^4} \cdot du$$

$$= -\frac{3}{4} \ln |5 - u^4| + C$$

$$= -\frac{3}{4} \ln |5 - (\sqrt[3]{x})^4| + C$$

3

$$\int \sqrt[3]{1 + x^{-\frac{3}{4}}} \cdot dx$$

* الحل : بسط ثم تعويض

$$\int \sqrt[3]{1 + \frac{1}{x^{\frac{3}{4}}}} \cdot dx$$

$$\textcircled{7} \int (x+1)^3 (x^2+2x-7)^5 \cdot dx$$

* الحل : تعويض

$$\int (x+1)^3 \cdot u^5 \cdot \frac{du}{2(x+1)}$$

$$\int (x^2+2x+1) \frac{u^5}{2} \cdot du$$

$$\int (u+7+1) \frac{u^5}{2} \cdot du$$

$$\int (u+8) \frac{u^5}{2} \cdot du$$

$$\int \left(\frac{1}{2} u^6 + 4u^5 \right) \cdot du$$

$$= \frac{1}{2} \cdot \frac{u^7}{7} + 4 \cdot \frac{u^6}{6} + C$$

$$= \frac{1}{14} (x^2+2x-7)^7 + \frac{2}{3} (x^2+2x-7)^6 + C$$

$$u = x^2 + 2x - 7$$

$$\frac{du}{dx} = 2x + 2$$

$$\frac{du}{dx} = 2(x+1)$$

$$\frac{du}{2(x+1)} = dx$$

$$u + 7 = x^2 + 2x$$

* تمرين 22

$$\textcircled{1} \int \frac{\sqrt[3]{7 + \tan x}}{5 - 5 \sin^2 x} \cdot dx$$

* الحل :- بسط ثم تعويض

$$\int \frac{\sqrt[3]{7 + \tan x}}{5(1 - \sin^2 x)} \cdot dx$$

$$\int \frac{\sqrt[3]{7 + \tan x}}{5 \cos^2 x} \cdot dx$$

$$\int \frac{1}{5} \sec^2 x \cdot (7 + \tan x)^{\frac{1}{3}} \cdot dx$$

$$\int \frac{1}{5} \sec^2 x (u)^{\frac{1}{3}} \cdot \frac{du}{\sec^2 x}$$

$$\int \frac{1}{5} u^{\frac{1}{3}} \cdot du$$

$$u = 7 + \tan x$$

$$\frac{du}{dx} = \sec^2 x$$

$$\frac{du}{\sec^2 x} = dx$$

$$\int (\sin x)^{\frac{1}{2}} \cdot \cos x, x \in [0, \frac{\pi}{2}]$$

$$\int u^{\frac{1}{2}} \cdot \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}}$$

$$\int u^{\frac{1}{2}} \cdot du$$

$$\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\frac{2}{3} (\sqrt{\sin x})^3 + C$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$\frac{du}{\cos x} = dx$$

* ترقية حل سؤال (3) ص 22

$$\int 3 \sqrt{\frac{x^{\frac{3}{4}} + 1}{x^{\frac{3}{4}}}} \cdot dx$$

$$\int \frac{\sqrt[3]{x^{\frac{3}{4}} + 1}}{\sqrt[3]{x^{\frac{3}{4}}}} \cdot dx$$

$$\int \frac{(x^{\frac{3}{4}} + 1)^{\frac{1}{3}}}{(x^{\frac{3}{4}})^{\frac{1}{3}}} \cdot dx$$

$$\int (x^{\frac{3}{4}} + 1)^{\frac{1}{3}} \cdot x^{-\frac{1}{4}} \cdot dx$$

$$\int u^{\frac{1}{3}} \cdot \cancel{x^{\frac{1}{4}}} \cdot \frac{4 \cdot du}{3 \cancel{x^{\frac{1}{4}}}}$$

$$\int \frac{4}{3} u^{\frac{1}{3}} \cdot du$$

$$\frac{4}{3} \cdot \frac{u^{\frac{4}{3}}}{\frac{4}{3}} + C$$

$$= (3 \sqrt[3]{u})^4 + C$$

$$= (3 \sqrt[3]{x^{\frac{3}{4}} + 1})^4 + C$$

$$u = x^{\frac{3}{4}} + 1$$

$$\frac{du}{dx} = \frac{3}{4} x^{-\frac{1}{4}}$$

$$\frac{4 du}{3 x^{-\frac{1}{4}}} = dx$$

$$\textcircled{5} \int e^{\sin x + \ln \cos x} \cdot dx$$

* الحل : بسط ثم تبسيط

$$\int e^{\sin x} \cdot \cancel{e^{\ln \cos x}} \cdot dx$$

$$\int e^{\sin x} \cdot \cos x \cdot dx$$

$$\int e^u \cdot \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}}$$

$$\int e^u \cdot du$$

$$= e^u + C$$

$$= e^{\sin x} + C$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$\frac{du}{\cos x} = dx$$

$$\textcircled{4} \int \sqrt{\sin x - \sin^3 x} \cdot dx$$

* الحل : بسط ثم تبسيط

$$\int \sqrt{\sin x (1 - \sin^2 x)} \cdot dx$$

$$\int \sqrt{\sin x \cdot \cos^2 x} \cdot dx$$

$$\int \sqrt{\sin x} \cdot \sqrt{\cos^2 x} \cdot dx$$

* تمرین 23 :-

① $\int \cos^3 x \cdot dx$

$\int \cos x \cdot \cos^2 x \cdot dx$

$\int \cos x (1 - \sin^2 x) \cdot dx$

$\int \cancel{\cos x} (1 - u^2) \cdot \cancel{\frac{du}{\cos x}}$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\frac{du}{\cos x} = dx$

$\int (1 - u^2) \cdot du$

$u - \frac{u^3}{3} + C$

$\sin x - \frac{1}{3}(\sin x)^3 + C$

② $\int \cot^3 x \cdot dx$

$\int \cot x \cdot \cot^2 x \cdot dx$

$\int \cot x (\csc^2 x - 1) \cdot dx$

$\int (\cot x \csc^2 x - \cot x) \cdot dx$

$\int \cot x \csc^2 x \cdot dx - \int \cot x \cdot dx$

تجویزاً و مباشرتاً

$-\frac{1}{2} \cot^2 x - \int \frac{\cos x}{\sin x} \cdot dx$

$-\frac{1}{2} \cot^2 x - \ln |\sin x| + C$

⑥ $\int \frac{5}{(4x^2 - 20x + 25)^9} \cdot dx$

* الحلال: تجویز

$\int \frac{5}{((2x-5)(2x-5))^9} \cdot dx$

$\int \frac{5}{((2x-5)^2)^9} \cdot dx$

$\int \frac{5}{(2x-5)^{18}} \cdot dx$

$\int 5 (2x-5)^{-18} = 5 \frac{(2x-5)^{-17}}{-17 \cdot 2} + C$

$= -\frac{5}{34} (2x-5)^{-17} + C$

⑦ $\int \frac{x^3}{(x+1)^5} \cdot dx$

* الحلال: مشتق ضربی معکوس

$\int \frac{x^3}{x^2(1+\frac{1}{x})^5} \cdot dx$

$\int \frac{1}{x^2} \cdot (1+\frac{1}{x})^{-5} \cdot dx$

$\int \frac{1}{x^2} \cdot u^{-5} \cdot -x^{-2} \cdot du$

$u = 1 + \frac{1}{x}$

$\frac{du}{dx} = -\frac{1}{x^2}$

$-x^2 \cdot du = dx$

$\int -u^{-5} \cdot du$

$-\frac{u^{-4}}{-4} + C$

$\frac{1}{4} (1 + \frac{1}{x})^{-4} + C$

* تصحيح 22 :-

$$\textcircled{9} \int \frac{(\sin x + \cos x)^5}{\sin^7 x} dx$$

$$\int \frac{\cancel{\sin^5 x} \left(1 + \frac{\cos x}{\sin x}\right)^5}{\cancel{\sin^5 x} \cdot \sin^2 x} dx$$

$$\int (1 + \cot x)^5 \csc^2 x dx$$

تعويض: ما داخل القوس بدونه بقوم

$$\int u^5 \cdot \cancel{\csc x} \cdot \frac{du}{-\cancel{\csc^2 x}} \quad \left\{ \begin{array}{l} u = 1 + \cot x \\ \frac{du}{dx} = -\csc^2 x \end{array} \right.$$

$$\int -u^5 \cdot du$$

$$= -\frac{u^6}{6} + C$$

$$= -\frac{(1 + \cot x)^6}{6} + C$$

$$\textcircled{8} \int \frac{(x^3 + 1)^5}{x^{19}} dx$$

الحل :- نجهز

(نضع x^3 مشترك
منه 8 شيء، البسط)

$$\int \frac{(x^3 (1 + \frac{1}{x^3}))^5}{x^{19}} dx$$

$$\int \frac{\cancel{x^{15}} (1 + x^{-3})^5}{\cancel{x^{15}} \cdot x^4} dx$$

$$\int \frac{(1 + x^{-3})^5}{x^4} dx$$

تعويض (ما داخل القوس)

$$\int \frac{u^5}{\cancel{x^4}} \cdot \frac{du}{-\cancel{3x^{-4}}} \quad \left\{ \begin{array}{l} u = 1 + x^{-3} \\ \frac{du}{dx} = -3x^{-4} \end{array} \right.$$

$$\int \frac{-1}{3} u^5 \cdot du$$

$$= -\frac{1}{3} \frac{u^6}{6} + C$$

$$= -\frac{1}{18} (1 + x^{-3})^6 + C$$

* تمرين (2) -

$$\textcircled{1} \int_0^{\frac{\pi}{3}} \sec^2 x \tan^5 x \cdot dx$$

$$\int_0^{\sqrt{3}} \cancel{\sec^2 x} \cdot u^5 \cdot \frac{du}{\cancel{\sec^2 x}}$$

$$\int_0^{\sqrt{3}} u^5 \cdot du$$

$$\frac{u^6}{6} \Big|_0^{\sqrt{3}}$$

$$\frac{1}{6} ((\sqrt{3})^6 - 0)$$

$$= \frac{1}{6} (27) = \boxed{\frac{9}{2}}$$

$$\begin{aligned} u &= \tan x \\ \frac{du}{dx} &= \sec^2 x \\ \frac{du}{\sec^2 x} &= dx \end{aligned}$$

$$\begin{aligned} x = \frac{\pi}{3} &\rightarrow u = \sqrt{3} \\ x = 0 &\rightarrow u = 0 \end{aligned}$$

* تمرين (1) 24 p

$$\textcircled{1} \int \sin^3 x \cos^9 x \cdot dx$$

$$\int \sin^2 x \cdot u^9 \cdot \frac{du}{-\sin x}$$

$$\int -(1 - \cos^2 x) u^9 \cdot du$$

$$\int (\cos^2 x - 1) u^9 \cdot du$$

$$\int (u^2 - 1) u^9 \cdot du$$

$$\int (u^{11} - u^9) \cdot du$$

$$= \frac{u^{12}}{12} - \frac{u^{10}}{10} + C$$

$$\frac{1}{12} (\cos x)^{12} - \frac{1}{10} (\cos x)^{10} + C$$

$$\begin{aligned} u &= \cos x \\ \frac{du}{dx} &= -\sin x \\ \frac{du}{-\sin x} &= dx \end{aligned}$$

$$\textcircled{2} \int \sin x \sec^5 x \cdot dx$$

* الحل: بسط التمام

$$\int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos^4 x} \cdot dx$$

$$\int \tan x \cdot \sec^4 x \cdot dx$$

$$\int \cancel{\tan x} \cdot u^4 \cdot \frac{du}{\cancel{\sec x \tan x}}$$

$$\int \frac{u^4}{u} \cdot du$$

$$\int u^3 \cdot du$$

$$\frac{u^4}{4} + C = \frac{1}{4} (\sec x)^4 + C$$

$$\begin{aligned} u &= \sec x \\ \frac{du}{dx} &= \sec x \tan x \\ \frac{du}{\sec x \tan x} &= dx \end{aligned}$$

$$\textcircled{2} \int \cot^3 x \csc^5 x \cdot dx$$

$$\int \cot^2 x \cdot u^5 \cdot \frac{du}{-\csc x \cot x}$$

$$\int \frac{(\csc^2 x - 1) u^5 \cdot du}{-\csc x}$$

$$\int \frac{(u^2 - 1) u^5}{-u} \cdot du$$

$$\int (1 - u^2) u^4 \cdot du$$

$$\int (u^4 - u^6) \cdot du$$

$$= \frac{u^5}{5} - \frac{u^7}{7} + C$$

$$\frac{1}{5} (\csc x)^5 - \frac{1}{7} (\csc x)^7 + C$$

$$\begin{aligned} u &= \csc x \\ \frac{du}{dx} &= -\csc x \cot x \\ \frac{du}{-\csc x \cot x} &= dx \end{aligned}$$

-: 24 p. *
تحياتي *

$$\textcircled{3} \int \cot^7 x \csc^4 x dx$$

حل: *
تحويل

$$\int u^7 \cdot \csc^4 x \cdot \frac{du}{-\csc^2 x}$$

$$\int -u^7 \csc^2 x \cdot du$$

متطابقة
 $\csc^2 x = \cot^2 x + 1$

$$\int -u^7 (u^2 + 1) du$$

$$\int (-u^9 - u^7) du$$

$$= -\frac{u^{10}}{10} - \frac{u^8}{8} + C$$

جواب

$$= -\frac{\cot^{10} x}{10} - \frac{\cot^8 x}{8} + C$$

$$\textcircled{3} \int \frac{\sin x + \tan x}{\cos^3 x} dx$$

* اجل :- نوز

$$= \int \frac{\sin x}{\cos^3 x} dx + \int \frac{\tan x}{\cos^3 x} dx$$

$$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos^2 x} + \int \tan x \sec^3 x dx$$

توین

$$\int \tan x \sec^2 x \cdot dx$$

توینا تو مباش

$$\int y \cdot \cancel{\sec^2 x} \cdot \frac{dy}{\cancel{\sec^2 x}}$$

$$\int y \cdot dy$$

$$\frac{y^2}{2} + C$$

$$\frac{(\tan x)^2}{2}$$

$$\begin{aligned} y &= \tan x \\ \frac{dy}{dx} &= \sec^2 x \\ \frac{dy}{\sec^2 x} &= dx \end{aligned}$$

$$\int \cancel{\tan x} \cdot u^3 \cdot \frac{du}{\cancel{\sec x \tan x}}$$

$$\int \frac{u^3}{u} \cdot du$$

$$\int u^2 \cdot du$$

$$\frac{u^3}{3} + C$$

$$\frac{1}{3} (\sec x)^3 + C$$

$$\begin{aligned} u &= \sec x \\ \frac{du}{dx} &= \sec x \cdot \tan x \\ \frac{du}{\sec x \tan x} &= dx \end{aligned}$$

$$= \frac{1}{2} (\tan x)^2 + \frac{1}{3} (\sec x)^3 + C$$