

$$1) \int_5^6 \frac{3x-10}{x^2-7x+12} dx$$

$$\frac{3x-10}{x^2-7x+12} = \frac{A}{x-3} + \frac{B}{x-4}$$

$$= \int_5^6 \frac{1}{x-3} dx + \int_5^6 \frac{2}{x-4} dx$$

$$= \ln|x-3| \Big|_5^6 + 2 \ln|x-4| \Big|_5^6$$

$$3x-10 = A(x-4) + B(x-3)$$

$$x=4 \rightarrow 2 = 0 + B \rightarrow \boxed{B=2}$$

$$x=3 \rightarrow -1 = -A + 0 \rightarrow \boxed{A=1}$$

$$= \ln 3 - \ln 2 + 2(\ln 2 - \ln 1)$$

$$= \ln 3 - \ln 2 + 2 \ln 2$$

$$= \ln 3 + \ln 4 - \ln 2 = \ln \frac{12}{2} = \ln 6$$

$$2) \int \frac{2x-4}{(x^2+4)(x+2)} dx$$

$$\frac{2x-4}{(x^2+4)(x+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$$

$$= \int \frac{-1}{x+2} dx + \int \frac{x}{x^2+4} dx$$

$$= -\ln|x+2| + \frac{1}{2} \ln|x^2+4| + C$$

$$2x-4 = A(x^2+4) + (Bx+C)(x+2)$$

$$x=-2 \rightarrow -8 = 8A + 0 \rightarrow \boxed{A=-1}$$

$$x=0 \rightarrow -4 = -4 + (0+C)(2) \rightarrow \boxed{C=0}$$

$$x=1 \rightarrow -2 = -5 + B(3) \rightarrow \boxed{B=1}$$

$$4) \int \frac{3x^3+2x^2+12}{x^4+6x^2} dx$$

$$\frac{3x^3+2x^2+12}{x^4+6x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+6}$$

$$= \int \frac{2}{x^2} dx + \int \frac{3x}{x^2+6} dx$$

$$= \int 2x^{-2} dx + \int \frac{3x}{x^2+6} dx$$

$$= \frac{-2}{x} + \frac{3}{2} \ln|x^2+6| + C$$

$$3x^3+2x^2+12 = A(x)(x^2+6) + B(x^2+6) + (Cx+D)(x^2)$$

$$x=0 \rightarrow 12 = 6B \rightarrow \boxed{B=2}$$

$$x=1 \rightarrow 17 = 7A + 14 + (C+D)(1)$$

$$x=-1 \rightarrow 11 = -7A + 14 + (-C+D)(1)$$

جمع المعادلتين

$$28 = 28 + 2D \rightarrow \boxed{D=0}$$

$$2D=0$$

$$x=2 \quad 44 = 20A + 20 + (2C)(4)$$

$$24 = 20A + 8C \rightarrow 6 = 5A + 2C$$

$$C = 3 - \frac{5}{2}A$$

$$17 = 7A + 14 + 3 - \frac{5}{2}A$$

$$0 = \frac{9}{2}A \rightarrow \boxed{A=0}$$

$$\boxed{C=3}$$

\* تمرين 34 : -

3  $\int \frac{2x^2 + 9x - 11}{x^3 + 2x^2 - 5x - 6} dx$

\* الحل : قسمة تركيبية للحصاف

$$x^3 + 2x^2 - 5x - 6$$

نحرب زفار

$$x = -1 \rightarrow -1 + 2 + 5 - 6$$

$$1 + 5 - 6$$

$$6 - 6 = 0$$

$$x = -1$$

قسمة تركيبية

	$x^3$	$x^2$	$x^1$	$x^0$
	1	2	-5	-6
$\downarrow$		-1	-1	6
	1	1	-6	0

$$(x+1)(x^2+x-6)$$

$$\int \frac{2x^2 + 9x - 11}{(x+1)(x^2+x-6)} dx$$

$$\int \frac{2x^2 + 9x - 11}{(x+1)(x+3)(x-2)} dx$$

$$= \int \frac{A}{x+1} \cdot dx + \int \frac{B}{x+3} \cdot dx + \int \frac{C}{x-2}$$

$$2x^2 + 9x - 11 = A(x-2)(x+3) + B(x+1)(x-2) + C(x+1)(x+3)$$

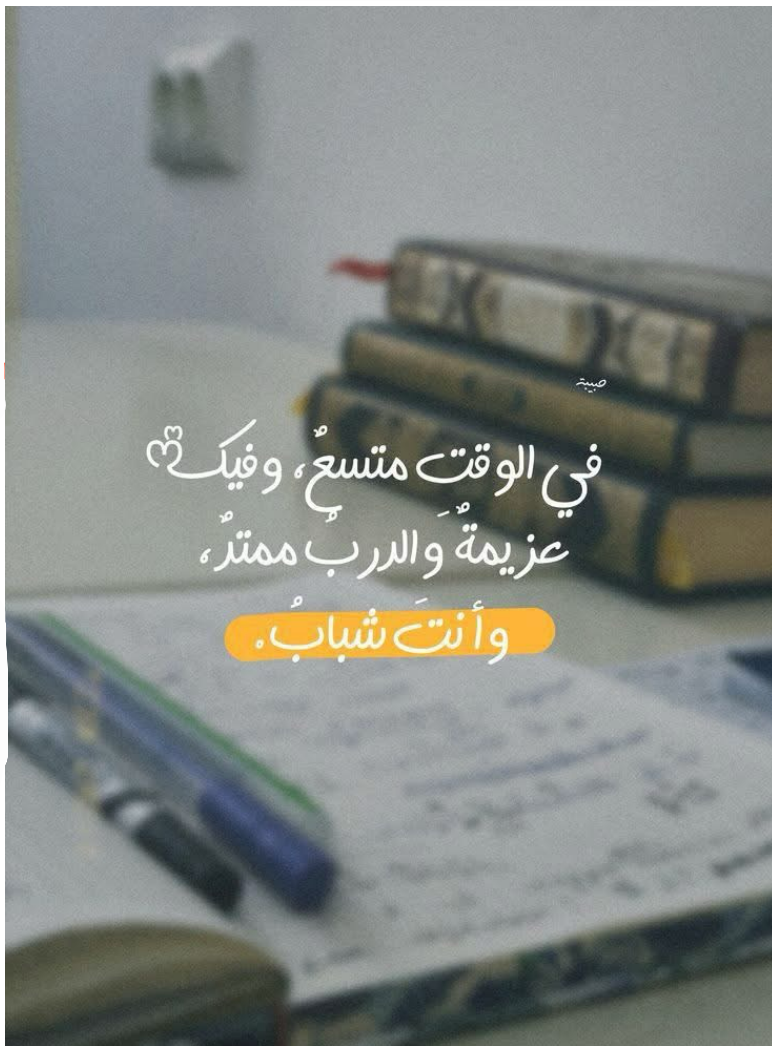
\*  $x = -1 \rightarrow 2 - 9 - 11 = A(-3)(2)$   
 $-18 = -6A \rightarrow 3 = A$

\*  $x = -3 \rightarrow 18 - 27 - 11 = B(-2)(-5)$   
 $-20 = 10B \rightarrow -2 = B$

\*  $x = 2 \rightarrow 8 + 18 - 11 = C(3)(5)$   
 $15 = 15C \rightarrow 1 = C$

$$= \int \frac{3}{x+1} dx + \int \frac{-2}{x+3} dx + \int \frac{1}{x-2} dx$$

$$3 \ln |x+1| - 2 \ln |x+3| + \ln |x-2| + C$$



$$1) \int \frac{4x^3 - 5}{2x^2 - x - 1} dx$$

قسمة طويلة

$$\begin{array}{r} 2x+1 \\ 2x^2-x-1 \overline{) 4x^3-5} \\ \underline{-4x^3+2x^2+2x} \phantom{-5} \\ 2x^2+2x-5 \\ \underline{-2x^2+x+1} \\ 3x-4 \end{array}$$

$$= \int (2x+1 + \frac{3x-4}{2x^2-x-1}) dx$$

الباقى 3x-4

$$= \int (2x+1 + \frac{11}{3} (\frac{1}{2x+1}) + \frac{-1}{3} (\frac{1}{x-1})) dx$$

$$\frac{3x-4}{2x^2-x-1} = \frac{A}{2x+1} + \frac{B}{x-1}$$

$$= x^2 + x + \frac{11}{6} \ln|2x+1| + \frac{-1}{3} \ln|x-1| + C$$

$$3x-4 = A(x-1) + B(2x+1)$$

$$x=1 \rightarrow -1 = 3B \rightarrow B = -\frac{1}{3}$$

$$x = -\frac{1}{2} \rightarrow -\frac{11}{2} = \frac{-3A}{2} \rightarrow A = \frac{11}{3}$$

$$2) \int_{-1/3}^{1/3} \frac{9x^2+4}{9x^2-4} dx$$

قسمة طويلة

$$= \int_{-1/3}^{1/3} (1 + \frac{8}{9x^2-4}) dx$$

القاعدة الخفراء

$$= \int_{-1/3}^{1/3} (1 + \frac{2}{3x-2} + \frac{-2}{3x+2}) dx$$

$$= \frac{2}{3} + \frac{2}{3} \ln|3x-2| \Big|_{-1/3}^{1/3} - \frac{2}{3} \ln|3x+2| \Big|_{-1/3}^{1/3}$$

$$= \frac{2}{3} + \frac{2}{3} (\ln(1) - \ln 3) - \frac{2}{3} (\ln 3 - \ln(1))$$

$$= \frac{2}{3} + \frac{-2}{3} \ln 3 - \frac{2}{3} \ln 3 = \frac{2}{3} - \frac{4}{3} \ln 3$$

$$\begin{array}{r} 1 \\ 9x^2-4 \overline{) 9x^2+4} \\ \underline{-9x^2+4} \\ 8 \end{array}$$

الباقى 8

$$\frac{8}{9x^2-4} = \frac{A}{3x-2} + \frac{B}{3x+2}$$

$$8 = A(3x+2) + B(3x-2)$$

$$x = \frac{2}{3} \quad 4A = 8 \quad A = 2$$

$$x = -\frac{2}{3} \quad -4B = 8 \quad B = -2$$

حل تمرين صفحة (34) (العوار الأيسر)

$$3) \int \frac{x^3 - 4x^2 - 2}{x^3 + x^2} dx \rightarrow \text{قسمة طويلة}$$

$$= \int (1 + \frac{-5x^2 - 2}{x^3 + x^2}) dx$$

$$= \int (1 + \frac{2}{x} + \frac{-2}{x^2} + \frac{-7}{x+1}) dx$$

$$= x + 2 \ln|x| + \frac{2}{x} - 7 \ln|x+1| + C$$

$$\begin{array}{r} 1 \\ x^3 + x^2 \overline{) x^3 - 4x^2 - 2} \\ \underline{-x^3 + x^2} \phantom{-2} \\ -5x^2 - 2 \end{array}$$

الباقي  $-5x^2 - 2$

$$\frac{-5x^2 - 2}{x^3 + x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$-5x^2 - 2 = A(x)(x+1) + B(x+1) + C(x^2)$$

$$x=0 \rightarrow -2 = B + 0 + 0 \quad \boxed{B=-2}$$

$$x=-1 \rightarrow -7 = 0 + 0 + C \quad \boxed{C=-7}$$

$$x=1 \rightarrow -7 = 2A + -4 + -7 \quad \boxed{A=2}$$

حل تمرين صفحة (37)

$$1) \int_0^{\pi/2} \frac{\cos x}{8 + \cos^2 x} dx = \int_0^{\pi/2} \frac{\cos x}{8 + 1 - \sin^2 x} dx = \int_0^{\pi/2} \frac{\cos x}{9 - \sin^2 x} dx$$

$$= \int_0^1 \frac{\cos x}{9 - y^2} \frac{dy}{(\cos x)} = \int_0^1 \frac{1}{9 - y^2} dy$$

$$= \int_0^1 \left( \frac{1/6}{3-y} + \frac{1/6}{3+y} \right) dy$$

$$= \frac{1}{6} \ln|3-y| \Big|_0^1 + \frac{1}{6} \ln|3+y| \Big|_0^1$$

$$= \frac{1}{6} (\ln 2 - \ln 3) + \frac{1}{6} (\ln 4 - \ln 3)$$

$$= \frac{1}{6} \ln 2 + \frac{1}{6} \ln 3 + \frac{1}{6} \ln 4 - \frac{1}{6} \ln 3$$

$$= \frac{1}{6} \ln 4 - \frac{1}{6} \ln 2 = \frac{1}{6} \ln 2$$

$y = \sin x \rightarrow \frac{dy}{\cos x} = dx$

$$\frac{1}{9 - y^2} = \frac{A}{3-y} + \frac{B}{3+y}$$

$$1 = A(3+y) + B(3-y)$$

$$y=3 \rightarrow 1 = 6A \quad \boxed{A=\frac{1}{6}}$$

$$y=-3 \rightarrow 1 = 6B \quad \boxed{B=\frac{1}{6}}$$

$y = \sin x$   
 $y|_0 = 0$   
 $x=0$   
 $y|_1 = 1$   
 $x = \frac{\pi}{2}$

تبدل حدود التكامل

$$2) \int \frac{\sec^2 x}{\tan^2 x - 1} dx$$

$$= \int \frac{\sec^2 x}{y^2 - 1} \frac{dy}{(\sec^2 x)} = \int \frac{1}{y^2 - 1} dy$$

$$= \int \left( \frac{\frac{1}{2}}{y-1} + \frac{\frac{1}{2}}{y+1} \right) dy$$

$$= \frac{1}{2} \ln|y-1| - \frac{1}{2} \ln|y+1| + C$$

$$= \frac{1}{2} \ln|\tan x - 1| - \frac{1}{2} \ln|\tan x + 1| + C$$

$$y = \tan x \rightarrow \frac{dy}{\sec^2 x} = dx$$

$$\frac{1}{y^2 - 1} = \frac{A}{y-1} + \frac{B}{y+1}$$

$$1 = A(y+1) + B(y-1)$$

$$y=1 \rightarrow A = \frac{1}{2}$$

$$y=-1 \rightarrow B = -\frac{1}{2}$$

$$3) \int \frac{e^x}{e^{2x} - e^x} dx$$

$$= \int \frac{1}{y^2 - y} dy = \int \frac{1}{y(y-1)} dy$$

$$= \int \left( \frac{-1}{y} + \frac{1}{y-1} \right) dy$$

$$= -\ln|y| + \ln|y-1| + C$$

$$= -\ln|e^x| + \ln|e^x - 1| + C$$

$$= -\ln(e^x) + \ln|e^x - 1| + C$$

$$y = e^x \rightarrow \frac{dy}{e^x} = dx$$

$$\frac{1}{y^2 - y} = \frac{A}{y} + \frac{B}{y-1}$$

$$1 = A(y-1) + B(y)$$

$$y=0 \rightarrow -A=1 \quad A=-1$$

$$y=1 \rightarrow B=1$$

\* تشریح صفحہ (37) :-

$$(4) \int \frac{\csc^2 x}{2 - \csc^2 x} dx$$

\* الحل :- مطابقت الحقا  $\csc^2 x = \cot^2 x + 1$

$$\int \frac{\csc^2 x}{2 - (\cot^2 x + 1)} dx$$

له اجزا السالب  
(حقیقہ)

$$= \int \frac{\csc^2 x}{2 - \cot^2 x - 1} dx$$

الآن تعریف:

$$\begin{aligned} u &= \cot x \\ \frac{du}{dx} &= -\csc^2 x \\ \frac{du}{-\csc^2 x} &= dx \end{aligned}$$

$$= \int \frac{\csc^2 x}{1 - \cot^2 x} dx$$

$$= \int \frac{\cancel{\csc^2 x}}{1 - u^2} \cdot \frac{du}{-\cancel{\csc^2 x}}$$

$$= \int \frac{1}{u^2 - 1} du$$

الآن  
عسور جزئیات

$$\int \frac{1}{(u-1)(u+1)} du = \int \frac{A}{u-1} du + \int \frac{B}{u+1} du$$

$$1 = A(u+1) + B(u-1)$$

$$u=1 \rightarrow 1 = 2A + 0 \rightarrow \boxed{\frac{1}{2} = A}$$

$$u=-1 \rightarrow 1 = 0 + -2B \rightarrow \boxed{-\frac{1}{2} = B}$$

$$= \int \frac{\frac{1}{2}}{u-1} du + \int \frac{-\frac{1}{2}}{u+1} du$$

$$= \frac{1}{2} \ln|u-1| + \frac{-1}{2} \ln|u+1| + C$$

لاشی شرح  $u$  لا تشریح  $C$

$$= \frac{1}{2} \ln|\cot x - 1| - \frac{1}{2} \ln|\cot x + 1| + C$$

\* حل آخر :- نضرب  $\csc^2 x$  بـ

$$\int \frac{1}{\sin^2 x} dx = \int \frac{\frac{1}{\cancel{\sin^2 x}}}{\frac{2\sin^2 x - 1}{\cancel{\sin^2 x}}} dx$$

توحيد المقامات  
 $\frac{\sin^2 x \cdot 2}{\sin^2 x} - \frac{1}{\sin^2 x}$

$$= \int \frac{1}{2\sin^2 x - 1} dx$$

$\rightarrow$  المتطابقة الأولى  $-\cos 2x$

$$= \int \frac{1}{-\cos 2x} dx$$

$$= \int -\sec 2x \cdot dx \times \frac{\sec 2x + \tan 2x}{\sec 2x + \tan 2x}$$

$$= -\frac{1}{2} \int \frac{2(\sec^2 2x + \sec 2x \tan 2x)}{\sec 2x + \tan 2x} dx$$

$$= -\frac{1}{2} \ln |\sec 2x + \tan 2x| dx$$

"لاحظ الجواب يختلف قليلاً"

$$5) \int \frac{2x}{(x^2+1)(x^2+2)^2} dx$$

$$y = x^2 \quad dx = \frac{dy}{2x}$$

$$\int \frac{1}{(y+1)(y+2)^2} dy$$

$$\frac{1}{(y+1)(y+2)^2} = \frac{A}{y+1} + \frac{B}{y+2} + \frac{C}{(y+2)^2}$$

$$1 = A(y+2)^2 + B(y+1)(y+2) + C(y+1)$$

$$y = -1 \rightarrow 1 = A$$

$$y = -2 \rightarrow 1 = -C \rightarrow C = -1$$

$$y = 0 \rightarrow 1 = 4 + 2B - 1 \rightarrow B = -1$$

$$\int \left( \frac{1}{y+1} - \frac{1}{y+2} - \frac{1}{(y+2)^2} \right) dy$$

$$\ln|y+1| - \ln|y+2| + \frac{1}{y+2} + C$$

صوابين  
لوجاريتمات

$$\ln\left(\frac{x^2+1}{x^2+2}\right) + \frac{1}{x^2+2} + C$$

$$6) \int \frac{x}{16x^4-1} dx = \int \frac{x}{(4x^2)^2-1} dx$$

$$y = x^2 \quad dx = \frac{dy}{2x}$$

$$\int \frac{x}{(4y)^2-1} \frac{dy}{2x}$$

$$= \frac{1}{2} \int \frac{1}{16y^2-1} dy = \frac{1}{2} \int \frac{1}{(4y+1)(4y-1)} dy$$

$$\frac{1}{(4y+1)(4y-1)} = \frac{A}{4y+1} + \frac{B}{4y-1}$$

$$1 = A(4y-1) + B(4y+1)$$

$$y = \frac{1}{4} \rightarrow 1 = B2 \rightarrow B = \frac{1}{2}$$

$$y = 0 \rightarrow 1 = -A + \frac{1}{2} \rightarrow A = -\frac{1}{2}$$

$$\frac{1}{2} \int \left( \frac{-\frac{1}{2}}{4y+1} + \frac{\frac{1}{2}}{4y-1} \right) dy = \frac{1}{4} \int \left( \frac{-1}{4y+1} + \frac{1}{4y-1} \right) dy$$

$$\frac{1}{4} \left( -\frac{1}{4} \ln|4y+1| + \frac{1}{4} \ln|4y-1| \right) + C$$

$$\frac{1}{16} \ln\left(\frac{4x^2-1}{4x^2+1}\right) + C$$

على صوابين  
الوجاريتمات

### حل تمرين صفحة (39)

$$1) \int \frac{\cos(\ln x)}{x} dx$$

$$y = \ln x \rightarrow dy(x) = dx$$

$$= \int \frac{\cos y}{x} x dy = \int \cos y dy = \sin y + C = \sin(\ln x) + C$$

$$2) \int \ln(2x+1) dx$$

$$= x \ln(2x+1) - \int \frac{2x}{2x+1} dx$$

$$= x \ln(2x+1) - \int \left( 1 - \frac{1}{2x+1} \right) dx$$

$$= x \ln(2x+1) - \left( x - \frac{1}{2} \ln(2x+1) \right) + C$$

$$u = \ln(2x+1) \quad du = \frac{2}{2x+1} dx$$

$$du = dx$$

$$u = x$$

$$\begin{array}{r} 2x+1 \overline{) 2x} \\ \underline{-2x+1} \\ \phantom{2x+1} \end{array}$$

الباقى

□

$$3) \int \ln\left(\frac{1}{\sqrt[3]{x}}\right) dx = \int \ln(x^{-1/3}) dx = -\frac{1}{3} \int \ln x dx$$

$$= -\frac{1}{3} \left( x \ln x - \int x \frac{dx}{x} \right)$$

$$= -\frac{1}{3} (x \ln x - \int dx) = -\frac{1}{3} (x \ln x - x) + c$$

$u = \ln x$        $du = \frac{dx}{x}$   
 $dv = dx$        $v = x$

$$4) \int x (\ln x)^2 dx$$

$$= \frac{x^2}{2} (\ln x)^2 - \int x \ln x dx$$

$$= \frac{x^2}{2} (\ln x)^2 - \left( \frac{x^2}{2} \ln x - \int \frac{x}{2} dx \right)$$

$$= \frac{x^2}{2} (\ln x)^2 - \frac{x^2}{2} \ln x + \frac{x^2}{4} + c$$

$u = (\ln x)^2$        $du = 2(\ln x) \left(\frac{1}{x}\right)$   
 $dv = x dx$        $v = \frac{x^2}{2}$

$u = \ln x$        $du = \frac{dx}{x}$   
 $dv = x dx$        $v = \frac{x^2}{2}$

$$5) \int \sec^2 x \ln(\sin x) dx$$

$$= \tan x \ln(\sin x) - \int \cot x \tan x dx$$

$$= \tan x \ln(\sin x) - \int dx = \tan x \ln(\sin x) - x + c$$

$u = \ln(\sin x)$        $du = \frac{\cos x}{\sin x} dx$   
 $dv = \sec^2 x dx$        $v = \tan x$

$$6) \int \ln(x^2-1) dx$$

$$= x \ln(x^2-1) - \int \frac{2x}{x^2-1} dx$$

$$= x \ln(x^2-1) - \int \left( 2 + \frac{2}{x^2-1} \right) dx$$

$$= x \ln(x^2-1) - \int \left( 2 + \frac{1}{x-1} + \frac{-1}{x+1} \right) dx$$

$$= x \ln(x^2-1) - 2x - \ln|x-1| + \ln|x+1| + c$$

$u = \ln(x^2-1)$        $du = \frac{2x}{x^2-1} dx$   
 $dv = dx$        $v = x$

قسمة طويلة

$$\begin{array}{r} 2 \\ x^2-1 \overline{) 2x^2} \\ \underline{-2x^2+2} \\ 2 \end{array}$$

$$\frac{2}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$2 = A(x+1) + B(x-1)$$

$$x=1 \rightarrow A=1$$

$$x=-1 \rightarrow B=-1$$

$$7) \int \frac{1}{x((\ln(x))^2 - 3\ln(x) - 4)} dx$$

$$y = \ln x \rightarrow \frac{dy}{dx} = \frac{1}{x} \rightarrow (x)dy = dx$$

$$= \int \frac{1}{x(y^2 - 3y - 4)} (x) dy = \int \frac{1}{y^2 - 3y - 4} dy$$

$$= \int \left( \frac{\frac{1}{5}}{y-4} + \frac{-\frac{1}{5}}{y+1} \right) dy$$

$$\frac{1}{y^2 - 3y - 4} = \frac{A}{y-4} + \frac{B}{y+1}$$

$$= \frac{1}{5} \ln|y-4| + \frac{-1}{5} \ln|y+1| + C$$

$$1 = A(y+1) + B(y-4)$$

$$y=4 \rightarrow \boxed{A = \frac{1}{5}}$$

$$y=-1 \rightarrow \boxed{B = -\frac{1}{5}}$$

$$= \frac{1}{5} \ln|\ln x - 4| - \frac{1}{5} \ln|\ln x + 1| + C$$

$$8) \int \cot x \ln(\sin x) dx$$

$$y = \sin x \rightarrow \frac{dy}{\cos x} = dx$$

$$= \int \frac{\cos x}{\sin x} \ln(y) \frac{dy}{\cos x} = \int \frac{1}{\sin x} \ln(y) dy = \int \frac{1}{y} \ln(y) dy$$

$$= \frac{(\ln y)^2}{2} + C = \frac{(\ln(\sin x))^2}{2} + C$$

تحويل مباشر

$$9) \int \frac{\tan x}{(\ln(\cos x))^2 - 25} dx$$

$$y = \ln(\cos x) \rightarrow \frac{dy}{dx} = \frac{-\sin x}{\cos x}$$

$$\frac{dy}{-\tan x} = dx$$

$$= \int \frac{\tan x}{y^2 - 25} \frac{dy}{-\tan x}$$

$$= \int \frac{1}{25 - y^2} dy$$

$$\frac{1}{25 - y^2} = \frac{A}{5-y} + \frac{B}{5+y}$$

$$1 = A(5+y) + B(5-y)$$

$$y=5 \rightarrow \boxed{A = \frac{1}{10}}$$

$$y=-5 \rightarrow \boxed{B = \frac{1}{10}}$$

$$= \int \left( \frac{\frac{1}{10}}{5-y} + \frac{\frac{1}{10}}{5+y} \right) dy$$

$$= -\frac{1}{10} \ln|5-y| + \frac{1}{10} \ln|5+y| + C$$

$$= -\frac{1}{10} \ln|5 - \ln(\cos x)| + \frac{1}{10} \ln|5 + \ln(\cos x)| + C$$

$$1) \int \frac{1}{1 - \sqrt[3]{x}} dx$$

$$y = \sqrt[3]{x} \rightarrow 3y^2 dy = dx$$

$$= \int \frac{3y^2}{1-y} dy$$

$$= \int \left( -3y - 3 + \frac{3}{1-y} \right) dy$$

$$= -\frac{3}{2} y^2 - 3y + 3 \ln|1-y| + C$$

$$= -\frac{3}{2} \sqrt[3]{x^2} - 3\sqrt[3]{x} - 3 \ln|1-\sqrt[3]{x}| + C$$

$$\begin{array}{r} -3y + 3 \\ 1-y \overline{) 3y^2} \\ \underline{+3y + 3y^2} \\ 3y \\ \underline{+3 + 3y} \end{array}$$

البقي 3

$$2) \int \frac{1}{\sqrt{1-\sqrt{x}}} dx$$

$$y = 1 - \sqrt{x} \rightarrow \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$(2\sqrt{x}) dy = dx$$

$$= \int \frac{-2\sqrt{x}}{\sqrt{y}} dy = \int \frac{2y-2}{\sqrt{y}} dy = \int \left( 2\sqrt{y} - \frac{2}{\sqrt{y}} \right) dy$$

$$= \int (2y^{1/2} - 2y^{-1/2}) dy = 2 \frac{(2y)^{3/2}}{3} - 2(2y)^{1/2} + C$$

$$= \frac{4}{3} \sqrt{y^3} - 4\sqrt{y} + C = \frac{4}{3} \sqrt{(1-\sqrt{x})^3} - 4\sqrt{1-\sqrt{x}} + C$$

$$3) \int_1^9 \frac{\sqrt{x}}{x-4} dx$$

$$y = \sqrt{x} \quad y^2 = x \quad 2y dy = dx$$

$$= \int_1^3 \frac{y(2y)}{y^2-4} dy = \int_1^3 \frac{2y^2}{y^2-4} dy$$

$$\begin{array}{r} 2 \\ y^2-4 \overline{) 2y^2} \\ \underline{-2y^2+8} \\ 8 \end{array}$$

$$\begin{array}{l} y = \sqrt{x} \\ x=1 \quad y=1 \\ x=9 \quad y=3 \end{array}$$

$$= \int_1^3 \left( 2 + \frac{8}{y^2-4} \right) dy = 2(2) + \int_1^3 \frac{8}{y^2-4} dy$$

$$= 4 + \int_1^3 \left( \frac{2}{y-2} + \frac{-2}{y+2} \right) dy$$

$$\frac{8}{y^2-4} = \frac{A}{y+2} + \frac{B}{y-2}$$

$$8 = A(y-2) + B(y+2)$$

$$y=2 \quad B=2$$

$$y=-2 \quad A=-2$$

$$3) \text{ تكملة } 4 + \int_1^3 \left( \frac{2}{y-2} + \frac{-2}{y+2} \right) dy$$

$$= 4 + 2 \ln|y-2| \Big|_1^3 - 2 \ln|y+2| \Big|_1^3$$

$$= 4 + 2(\ln 1 - \ln 1) - 2(\ln 5 - \ln 3) = 4 + 0 + -2 \ln \frac{5}{3}$$

$$= 4 - 2 \ln \frac{5}{3} = 4 + 2 \ln \frac{3}{5} = 4 + \ln \frac{9}{25}$$

$$4) \int \sqrt{1-e^x} dx$$

$$\int y \left( \frac{2y}{-e^x} \right) dy$$

$$= \int \frac{2y^2}{y^2-1} dy$$

$$= \int \left( 2 + \frac{2}{y^2-1} \right) dy$$

$$= \int \left( 2 + \frac{-1}{y+1} + \frac{1}{y-1} \right) dy$$

$$= 2y - \ln|y+1| + \ln|y-1| + C$$

$$= 2\sqrt{1-e^x} - \ln|\sqrt{1-e^x}+1| + \ln|\sqrt{1-e^x}-1| + C$$

$$y = \sqrt{1-e^x} \rightarrow y^2 = 1-e^x \rightarrow 2y \frac{dy}{dx} = -e^x$$

$$\frac{2y}{-e^x} dy = dx$$

$$\begin{array}{r} 2 \\ y^2-1 \overline{) 2y^2} \\ \underline{-2y^2+2} \\ 2 \end{array}$$

$$\frac{2}{y^2-1} = \frac{A}{y+1} + \frac{B}{y-1}$$

$$2 = A(y-1) + B(y+1)$$

$$y=1 \quad B=1$$

$$y=-1 \quad A=-1$$

$$5) \int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx$$

$$= \int \frac{6y^5}{y^3-y^2} dy = \int \frac{6y^3}{y-1} dy$$

$$= \int \left( 6y^2 + 6y + 6 + \frac{6}{y-1} \right) dy$$

$$= 2y^3 + 3y^2 + 6y + 6 \ln|y-1| + C$$

$$= 2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6 \ln|\sqrt[6]{x}-1| + C$$

$$y = \sqrt[6]{x}$$

$$y^6 = x$$

$$6y^5 dy = dx$$

$$\begin{array}{r} 6y^2+6y+6 \\ y-1 \overline{) 6y^3} \\ \underline{-6y^3+6y^2} \\ 6y^2 \\ \underline{-6y^2+6y} \\ 6y \\ \underline{-6y+6} \\ 6 \end{array}$$

\* حل تمرين كفة (42) :-

$$\int x^n e^{ax} dx$$

$$u = x^n \longrightarrow du = n x^{n-1} dx$$

$$dv = e^{ax} \longrightarrow v = \frac{e^{ax}}{a}$$

$$\frac{x^n \cdot e^{ax}}{a} - \int \frac{n}{a} \cdot x^{n-1} \cdot e^{ax} dx$$

$$\frac{x^n \cdot e^{ax}}{a} - \frac{n}{a} \int x^{n-1} \cdot e^{ax} dx$$

$$\downarrow$$
$$w = x^{n-1} \longrightarrow dw = (n-1) x^{n-2} dx$$

$$dv = e^{ax} \longrightarrow \frac{e^{ax}}{a}$$

$$\frac{x^n \cdot e^{ax}}{a} - \frac{n}{a} \left( \frac{x^{n-1} \cdot e^{ax}}{a} - \int \frac{n-1}{a} \cdot x^{n-2} \cdot e^{ax} dx \right)$$

$$\frac{x^n \cdot e^{ax}}{a} - \frac{n}{a} \left( \frac{x^{n-1} \cdot e^{ax}}{a} - \frac{n-1}{a} \int x^{n-2} e^{ax} dx \right)$$